Bermudan Swaption

Bermudan Swaption Introduction

- A Bermudan swaption is an option on a swap with predefined exercise schedules.
- A Bermudan swaption gives the holder the right but not the obligation to enter a swap at predefined dates.
- Bermudan swaptions give the holders some flexibility to enter swaps.
- A comparison of European, American and Bermudan swaptions
  - European swaption has only one exercise date at the maturity.
  - American swaption has multiple exercise dates (daily)
  - Bermudan swaption has multiple exercise dates (but not daily): such as quarterly, monthly, etc.

Payoffs

- At the maturity T, the payoff of a Bermudan swaption is given by

$$Payoff(T) = \max(0, V_{swap}(T))$$

where $V_{swap}(T)$ is the value of the underlying swap at T.

- At any exercise date T_i, the payoff of the Bermudan swaption is given by

$$Payoff(T_i) = max\left(V_{swap}(T_i), I(T_i)\right)$$

where $V_{swap}(T_i)$ is the exercise value of the Bermudan swap and $I(T_i)$ is the intrinsic value.

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Valuation

- Given the complexity of Bermudan swaption valuation, there is no closed form solution. Therefore, we need to select an interest rate term structure model and a numeric solution to price Bermudan swaptions numerically.
- The selection of interest rate term structure models
  - Popular interest rate term structure models:

    Hull-White, Linear Gaussian Model (LGM), Quadratic Gaussian Model (QGM), Heath Jarrow Morton (HJM), Libor Market Model (LMM).
  - HJM and LMM are too complex.
  - Hull-White is inaccurate for computing sensitivities.
  - Therefore, we choose either LGM or QGM.

Valuation (Cont.)

- The selection of numeric approaches
  - After selecting a term structure model, we need to choose a numeric approach to approximate the underlying stochastic process of the model.
  - Commonly used numeric approaches are tree, partial differential equation (PDE), lattice and Monte Carlo simulation.
  - Tree and Monte Carlo are notorious for inaccuracy on sensitivity calculation.
  - Therefore, we choose either PDE or lattice.

Valuation (Cont.)

- The dynamics of LGM

$$dX(t) = \alpha(t)dW$$

where X is the single state variable and W is the Wiener process.

- The numeraire is given by

$$N(t, X) = \left(H(t)X + 0.5H^2(t)\zeta(t)\right)/D(t)$$

- The zero coupon bond price is

$$B(t, X; T) = D(T)exp\left(-H(t)X - 0.5H^2(t)\zeta(t)\right)$$

Valuation (Cont.)

- The LGM model is mathematically equivalent to the Hull-White model but offers
  - Significant improvement of stability and accuracy for calibration.
  - Significant improvement of stability and accuracy for sensitivity calculation.
- The state variable is normally distributed under the appropriate measure.
- The LGM model has only one stochastic driver (one-factor), thus changes in rates are perfected correlated.

Valuation (Cont.)

- Match today's curve

  At time t=0, X(0)=0 and H(0)=0. Thus Z(0,0;T)=D(T). In other words, the LGM automatically fits today's discount curve.

- Select a group of market swaptions.
- Solve parameters by minimizing the relative error between the market swaption prices and the LGM model swaption prices.

Valuation (Cont.)

- Calibrate the LGM model.
- Create the lattice based on the LGM: the grid range should cover at least 3 standard deviations.
- Calculate the underlying swap value at each final note.
- Conduct backward induction process iteratively rolling back from final dates until reaching the valuation date.
- Compare exercise values with intrinsic values at each exercise date.
- The value at the valuation date is the price of the Bermudan swaption.

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Example

Swaption definition		
Counterparty	xxx	
Buy or sell	Sell	
Payer or receiver	Receiver	
Currency	USD	
Settlement	Cash	
Trade date	9/12/2012	
Underlying swap definition	Leg 1	Leg2
Day Count	dcAct360	dcAct360
Leg Type	Fixed	Float
Notional	250000	250000
Payment Frequency	1	1
Pay Receive	Receive	Pay
Start Date	9/14/2012	9/14/2012
End Date	9/14/2022	9/14/2022
Fix rate	0.0398	NA
Index Type	NA	LIBOR
Index Tenor	NA	1M
Index Day Count	NA	dcAct360
Exercise Schedules		
Exercise Type	Notification Date	Settlement Date
Call	1/12/2017	1/14/2017
Call	1/10/2018	1/14/2018

Reference:
https://finpricing.com/lib/EqWarrant.html